Comment on "Depinning Due to Quenched Randomness"

The statistical mechanics of interfaces in random media, as illustrated, for example, by disorder-induced critical wetting and roughening phenomena, has been a subject of great interest since the seminal work of Huse and Henley [1], Kardar and Nelson [2,3], and Lipowsky and Fisher [4]. The purpose of this Comment is to bring to resolution an outstanding issue that has remained at the heart of this matter for several years now. It concerns the liberation exponent ψ for interfacial depinning due to quenched randomness. One considers a 2D Ising model on a semi-infinite square lattice in which there is a energetically favorable row of weak bonds along the edge. The imposed boundary conditions are such that at low temperatures and in the absence of strong disorder there is a 1D elastic interface running through these weak bonds, localized by the contact pinning potential along the edge of the sample. As the quenched randomness in the bulk is augmented, however, the interface wanders increasingly further from the edge, searching for minimal energy configurations. Eventually it undergoes a depinning transition. Kardar [2], in his original work on the subject, used Bethe-ansatz techniques within a replica formalism to determine that the mean position of the 1D interface diverged *quadratically* as the disorder strength in the bulk was raised to its critical value. That $\psi=2$ for critical wetting in 2D random-bond Ising models can also be established via bead and necklace model ideas of Lipowsky and Fisher [4], who found that $\psi=\zeta/(1-\zeta)$, where $\zeta_{\rm RB}=\frac{2}{3}$ is the interfacial wandering exponent. Attempting to verify this novel surface critical phenomena, Kardar [2] performed a series of numerical simulations for disorder-induced depinning both from a wall and in the bulk. Somewhat paradoxically, though, his numerical efforts in this regard revealed only an *approximately linear* divergence, which he attributed to an inability to reach asymptotic scaling behavior. Examining this depinning transition using more advantageous scaling variables and with the benefit of somewhat greater computational power, we have managed to access the true critical regime, thereby observing the elusive quadratic divergence.

In Fig. 1, we show the results of our simulation, based on Kardar's solid-on-solid model in the presence of random bonds. In this formulation, the interface runs parallel to the x axis on a square lattice with $y\geq 1$, one end being pinned to $(0,1)$ and a particular configuration described by the set of integer heights $\{y(x)\}$. Whenever $y(x+1)=y(x)\pm 1$, the interface jumps vertically, incurring an elastic energy cost K, with associated Boltzmann weight $\gamma=\exp(-K)$. The horizontal segments along the edge $\mu(x,1)=\mu_e$ are weaker than those in the bulk, which are assumed to be independent random variables drawn uniformly from the interval $[\mu_a-s/2,\mu_a+s/2]$. The total Boltzmann weight of interfaces connecting $(0,1)$ to (x,y) is calculated recursively by transfer-matrix techniques [2]. We have used $\gamma=0.3$ and $\mu_a-\mu_e=0.34$, as in Kardar's original paper, and typically performed the disorder average over 500–1000 realizations of randomness. We estimate $s_c=1.54\pm0.01$. However, path lengths as large as 5.0×10^4, 7.5×10^4, 10^5 were necessary to achieve saturation for $s=1.45$, 1.47, 1.49, respectively. The figure shows the inverse localization length versus s^2, the latter being the most appropriate abscissa since the replica theory predicts $l^{-1}\sim(\text{const}-\kappa)^2$, where $\kappa^{-1}=48\gamma/s^2$. Note that although the quadratic behavior becomes apparent as $s^2\rightarrow s_c^2$, it is inevitably so only with the inclusion of the final three data points alluded to above, indicating the extreme difficulty of the problem. We find similar results for disorder-induced depinning in the bulk, but with a slightly larger critical regime. All this is in sharp contrast to the much less stubborn case of complete wetting in random media, for which it is considerably easier to elicit asymptotic scaling [5].

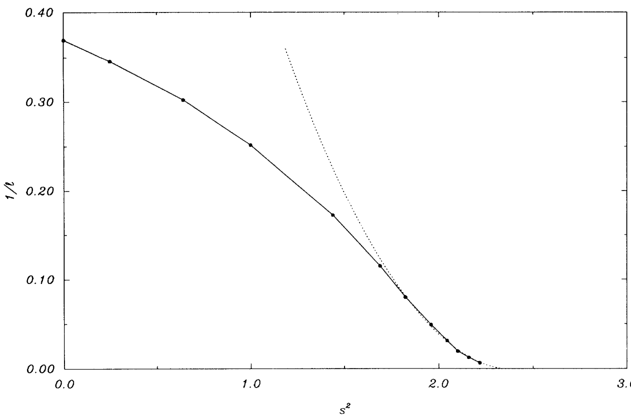

FIG. 1. Interfacial delocalization by quenched randomness from an edge contact potential. The inverse localization length, given by the reciprocal of the mean position of the interface, vanishes *quadratically* as the variance of the bulk disorder strength attains its critical value.

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